



SHENTON
COLLEGE

Year 12 Mathematics: Specialist Term 2 2020

Test 2 *Calculator Free* Functions, Graphs & Vectors in 3D

Student Name: _____

Teacher: Alfonsi Moore

Working Time: 30 minutes
Formula Sheet provided.

*Attempt **all** questions.
All necessary working and reasoning must be shown for **full marks**.*

Total Marks

37

Question 1.

(4 marks)

Solve the following system of linear equations using Gaussian elimination.

$$\begin{aligned}x - y + 2z &= 4 \\ -2x - y + 3z &= -1 \\ 4x - y - z &= 7\end{aligned}$$

Question 2.

(8 marks)

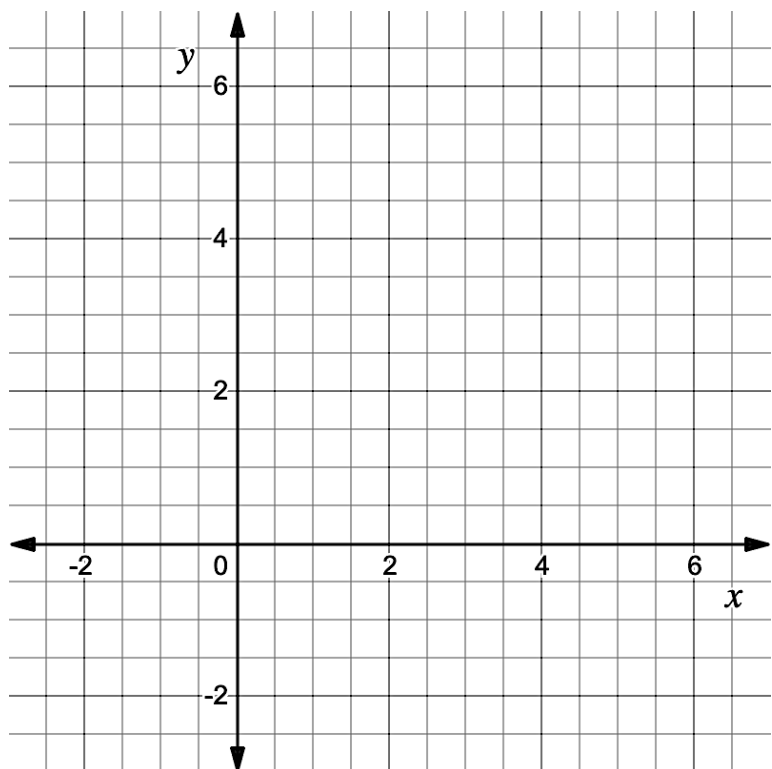
Two functions, f and g , are defined as:

$$f(x) = |x| + |2x + 2| \qquad g(x) = x - 3$$

(a) State $f(g(x))$, simplifying where possible. (1 mark)

(b) Hence, determine the piecewise definition of $f(g(x))$. (3 marks)

(c) Graph $y = f(g(x))$ on the axes provided below and hence, state its range. (3 marks)

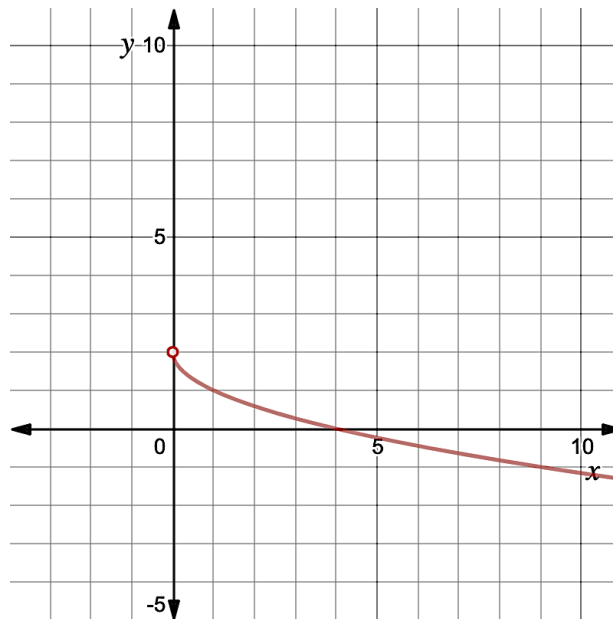


(d) Hence, or otherwise, solve $f(g(x)) \leq 2$. (1 mark)

Question 3.

(10 marks)

Consider the function $f(x) = k - \frac{x}{\sqrt{x}}$. The graph of $y = f(x)$ is shown below.



- (a) State the domain of $f(x)$. (1 mark)

- (b) State the value of k . (1 mark)

- (c) Show, algebraically, that $f(x)$ is indeed a one-to-one function. (2 marks)

- (d) Sketch $y = f^{-1}(x)$ on the same axes above and hence, solve $f(x) = f^{-1}(x)$. (3 marks)

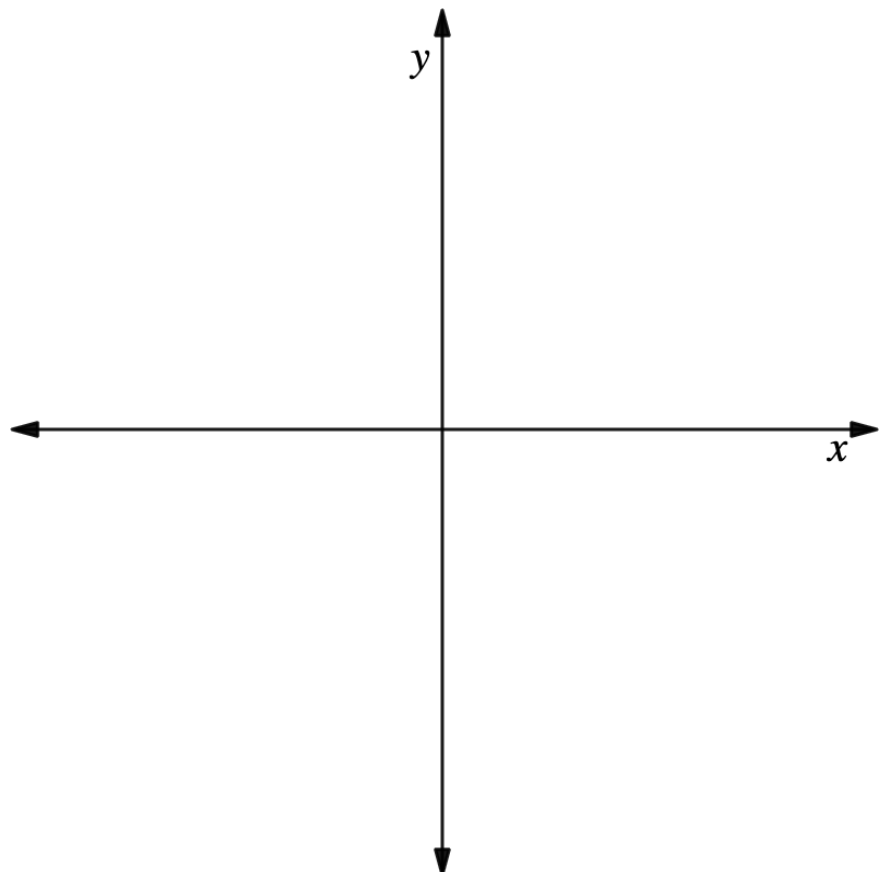
- (e) State the defining rule for $y = f^{-1}(x)$, including an appropriate restriction on the domain. (3 marks)

Question 4.

(8 marks)

Consider the rational function, $f(x) = \frac{x^3 - 2x^2 + x + 4}{x^2 - 4}$.

- (a) Sketch the rational function $y = f(x)$ on the axes provided below, labelling all critical points. *You do not need to locate the stationary points of the rational function.* (5 marks)

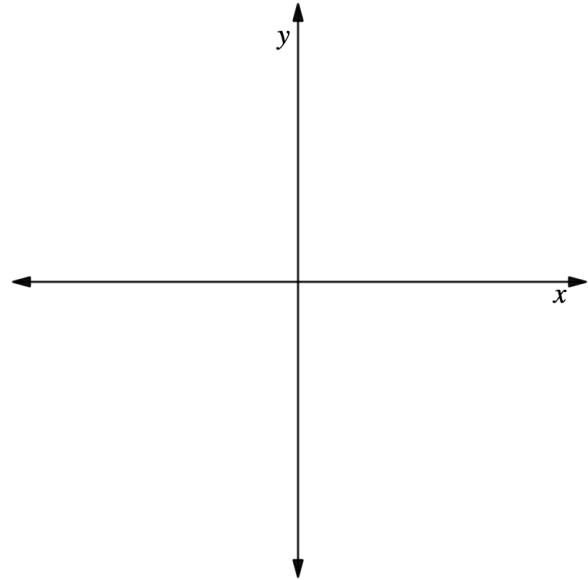


NOTE: For part (b), two sets of blank axes has been provided, however, there are no marks assigned to a sketch.

(b) Hence, determine the value(s) of x such that:

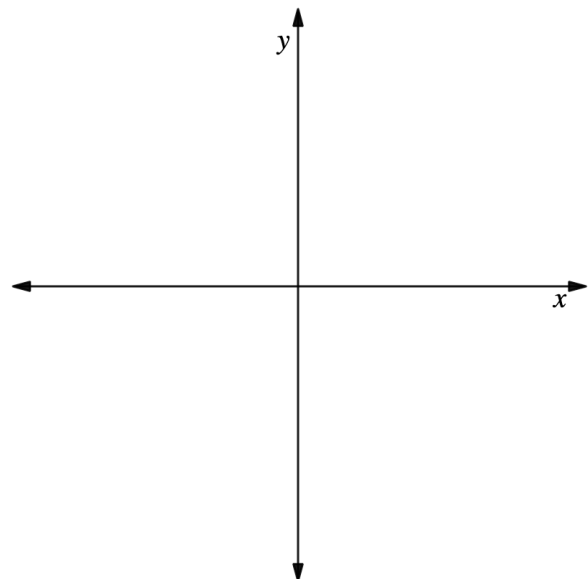
(i) $f(x) = |f(x)|$.

(2 marks)



(ii) $f(x) = f(|x|)$.

(1 mark)



Question 5.**(7 marks)**

A plane is defined by the vector equation $\Pi_1 : \mathbf{r} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -4 \\ 1 \\ 5 \end{pmatrix}$.

(a) Determine the equation of the plane in the form $\mathbf{r} \cdot \mathbf{n} = k$.

(3 marks)

A second plane has a Cartesian equation defined by $\Pi_2 : x + 2y + z = -2$.

Π_1 and Π_2 intersect along a line defined by the equation $ay + bz = d$, where $a, b, d \in \mathbb{Z}$.

(b) Determine a possible set of values for a, b and d .

(4 marks)

End of Calculator Free Section